MAIN RING MAGNET FIELD ERRORS

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In order to get a "feel" for the requirements on the fields in the main ring magnets, I have estimated the radial shifts resulting from errors in the magnetic field shape for several cases. For the quadrupoles, I have assumed that all effects are due to the errors in the horizontally focusing quadrupoles. I then solve approximately the equation

$$x'' + \nu_o^2 x + \alpha x^n = 0$$

where $\alpha = \frac{L_Q}{8 L_B} \frac{R}{B_o} n\, q_n$

and L_Q = length of a horizontally focusing quadrupole
L_B = length of a bending magnet
R = the machine radius (1000 m)
B_o = the field in a bending magnet
ν_o = the design betatron frequency

q_n is an error term coefficient from the expansion of the magnetic scalar potential in the quadrupole

$$V_m = \sum q_n r^n \sin n\Theta$$

The symmetry of the quadrupole causes all odd terms to vanish. The n=2 term gives the quadrupole field, so that the n=4 term (octupole) is the lowest error term. (This would vanish if the quadrupoles had true quadrupole symmetry.) The approximate formula for the ν shift is

$$\Delta\nu \simeq \frac{1}{2\nu_o} \frac{L_Q}{8 L_B} \frac{R}{B_o} \left(1 + \frac{n}{2}\right) q_n x_o^{n-2}$$

where x_o is the oscillation amplitude at β_{max}.

For the bending magnets, one solves the same equation, except that the definition of α is somewhat different,

$$\alpha = \frac{R}{B_o} \; n \; a_n$$

and the symmetry in this case causes all of the even terms in the expansion to vanish. $a_1 = B_o$ so that the first error term is the sextupole, n=3. The magnet calculations show that there is negligible sextupole error at fields up to 18,000 gauss, and the remaining errors look like decapole, n=5. If there is no momentum error, the first order effect of the field error vanishes and the ν shift is a second order effect.

$$\Delta \nu \sim \frac{\nu_o}{2} \left(\frac{R}{B_o} \; \frac{n \, a_n}{\nu_o^2} \; x_o^{n-2} \right)^2$$

To allow for the variation of amplitude with azimuth, the above result should be multiplied by

$$\left(\frac{\beta}{\beta_{max}} \right)_{AV}^{n-2}$$

The ν shifts calculated from this turn out to be very small. On the other hand, if there is a systematic radial displacement, δ , due to momentum error, there results an error term in the differential equation of the form

$$\alpha (x + \delta)^n = \alpha x^n + \alpha n x^{n-1} \delta + \cdots$$

The first term in the expansion gives the second order ν shift described above. The second term is the one of interest, because it is of opposite parity and therefore gives a first order shift

$$\Delta \nu \sim \frac{n\,(n+1)}{4\,\nu_o} \; \frac{R}{B_o} \; \alpha_n \; x_o^{n-3} \delta$$

Again, this should be multiplied by $\left(\frac{\beta}{\beta_{max}}\right)_{AV}^{\frac{n-3}{2}}$

to take into account the variation of amplitude with azimuth.

A few numerical examples will give some meaning to the magnitudes involved. In order to properly understand the results, it is worth noting that the extracted protons make three turns between the time they just clear the septum with x_o = 3 cm, and the time they pass through the channel. Since a 30^o phase change of the oscillation is sufficient to arrest the growth, the corresponding (average) ν shift of 0.028 would be a total disaster. In fact, for good extraction, we should allow less than half this value.

Quadrupole --- KASE 2630 (at 400 BeV)

		$\Delta \nu$	
n	α_n (kg-cm)	x_o = 3 cm	x_o = 4 cm
4	1.55×10^{-4}	+ .025	+ .045
6	-1.01×10^{-5}	- .020	- .062
8	4.24×10^{-7}	+ .009	+ .052

<u>Dipole</u> --- KASE 5122 (at 400 BeV)

n=5 $\alpha_5 \sim 2.8 \times 10^{-3}$ Gauss - cm

	$\Delta\nu$	
	x_o = 3 cm	x_o = 4 cm
Second Order	6.4×10^{-5}	3.6×10^{-4}
First Order (δ = 0.25 cm)	.0077	.014

It is clear that the quadrupole must be considerably improved. The first order effect due to momentum error could be largely eliminated by an adjustment of the bending magnet fields during the spill to keep the orbits directly leading to extraction centered. However this effect could cause a modulation of the spill due to ripple on the bending magnet current, so that the bending magnet errors must be minimized.